

6.24 The magnetic field in a dielectric material with $\epsilon = 4\epsilon_0$, $\mu = \mu_0$, and $\sigma = 0$ is given by

$$\mathbf{H}(y, t) = \hat{\mathbf{x}}5 \cos(2\pi \times 10^7 t + ky) \quad (\text{A/m}).$$

Find k and the associated electric field \mathbf{E} .

Solution: In phasor form, the magnetic field is given by $\tilde{\mathbf{H}} = \hat{\mathbf{x}}5e^{jky}$ (A/m). From Eq. (6.86),

$$\tilde{\mathbf{E}} = \frac{1}{j\omega\epsilon} \nabla \times \tilde{\mathbf{H}} = \frac{-jk}{j\omega\epsilon} \hat{\mathbf{z}}5 \exp jky$$

and, from Eq. (6.87),

$$\tilde{\mathbf{H}} = \frac{1}{-j\omega\mu} \nabla \times \tilde{\mathbf{E}} = \frac{-jk^2}{-j\omega^2\epsilon\mu} \hat{\mathbf{x}}5 \exp jky,$$

which, together with the original phasor expression for $\tilde{\mathbf{H}}$, implies that

$$k = \omega\sqrt{\epsilon\mu} = \frac{\omega\sqrt{\epsilon_r}}{c} = \frac{2\pi \times 10^7 \sqrt{4}}{3 \times 10^8} = \frac{4\pi}{30} \quad (\text{rad/m}).$$

Inserting this value in the expression for $\tilde{\mathbf{E}}$ above,

$$\tilde{\mathbf{E}} = -\hat{\mathbf{z}} \frac{4\pi/30}{2\pi \times 10^7 \times 4 \times 8.854 \times 10^{-12}} 5 \exp j4\pi y/30 = -\hat{\mathbf{z}}941 \exp j4\pi y/30 \quad (\text{V/m}).$$
