6.25 Given an electric field

$$\mathbf{E} = \hat{\mathbf{x}} E_0 \sin ay \cos(\omega t - kz),$$

where E_0, a, ω, and k are constants, find $\mathbf{H}$.

Solution:

$$\begin{aligned}
\mathbf{E} &= \hat{\mathbf{x}} E_0 \sin ay \cos(\omega t - kz), \\
\widetilde{\mathbf{E}} &= \hat{\mathbf{x}} E_0 \sin ay \, e^{-jkz}, \\
\widetilde{\mathbf{H}} &= -\frac{1}{j\omega\mu} \nabla \times \widetilde{\mathbf{E}} \\
&= -\frac{1}{j\omega\mu} \left[\hat{\mathbf{y}} \frac{\partial}{\partial z} (E_0 \sin ay \, e^{-jkz}) - \hat{\mathbf{z}} \frac{\partial}{\partial y} (E_0 \sin ay \, e^{-jkz}) \right] \\
&= \frac{E_0}{\omega\mu} [\hat{\mathbf{y}} k \sin ay - \hat{\mathbf{z}} \, ja \cos ay] e^{-jkz}, \\
\mathbf{H} &= \mathfrak{Re}[\widetilde{\mathbf{H}} e^{j\omega t}] \\
&= \mathfrak{Re} \left\{ \frac{E_0}{\omega\mu} [\hat{\mathbf{y}} k \sin ay + \hat{\mathbf{z}} a \cos ay \, e^{-j\pi/2}] e^{-jkz} e^{j\omega t} \right\} \\
&= \frac{E_0}{\omega\mu} \left[\hat{\mathbf{y}} k \sin ay \cos(\omega t - kz) + \hat{\mathbf{z}} a \cos ay \cos\left(\omega t - kz - \frac{\pi}{2}\right) \right] \\
&= \frac{E_0}{\omega\mu} [\hat{\mathbf{y}} k \sin ay \cos(\omega t - kz) + \hat{\mathbf{z}} a \cos ay \sin(\omega t - kz)] .
\end{aligned}$$