6.6 The square loop shown in Fig. P6.6 is coplanar with a long, straight wire carrying a current

$$I(t) = 5\cos(2\pi \times 10^4 t) \qquad \text{(A)}.$$

(a) Determine the emf induced across a small gap created in the loop.

(b) Determine the direction and magnitude of the current that would flow through a 4-Ω resistor connected across the gap. The loop has an internal resistance of 1 Ω.

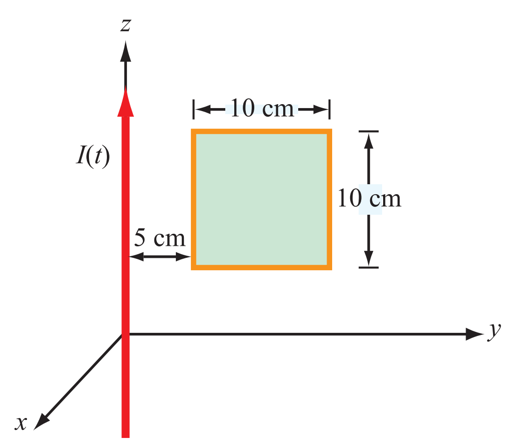

Figure P6.6 Loop coplanar with long wire (Problem 6.6).

Solution:

(a) The magnetic field due to the wire is

$$\mathbf{B} = \hat{\boldsymbol{\phi}}\,\frac{\mu_0 I}{2\pi r} = -\hat{\mathbf{x}}\,\frac{\mu_0 I}{2\pi y},$$

where in the plane of the loop, $\hat{\boldsymbol{\phi}} = -\hat{\mathbf{x}}$ and $r = y$. The flux passing through the loop is

$$\begin{aligned}
\Phi = \int_S \mathbf{B}\cdot d\mathbf{s} &= \int_{5\text{ cm}}^{15\text{ cm}} \left(-\hat{\mathbf{x}}\,\frac{\mu_0 I}{2\pi y}\right)\cdot[-\hat{\mathbf{x}}\,10\text{ (cm)}]\;dy \\
&= \frac{\mu_0 I \times 10^{-1}}{2\pi}\ln\frac{15}{5} \\
&= \frac{4\pi\times 10^{-7}\times 5\cos(2\pi\times 10^4 t)\times 10^{-1}}{2\pi}\times 1.1 \\
&= 1.1\times 10^{-7}\cos(2\pi\times 10^4 t) \qquad \text{(Wb)}.
\end{aligned}$$

$$
\begin{aligned}
V_{\text{emf}} = -\frac{d\Phi}{dt} &= 1.1 \times 2\pi \times 10^4 \sin(2\pi \times 10^4 t) \times 10^{-7} \\
&= 6.9 \times 10^{-3} \sin(2\pi \times 10^4 t) \qquad \text{(V)}.
\end{aligned}
$$

(b)

$$
I_{\text{ind}} = \frac{V_{\text{emf}}}{4+1} = \frac{6.9 \times 10^{-3}}{5} \sin(2\pi \times 10^4 t) = 1.38 \sin(2\pi \times 10^4 t) \qquad \text{(mA)}.
$$

At $t = 0$, $\mathbf{B}$ is a maximum, it points in the $-\hat{\mathbf{x}}$ direction, and since it varies as $\cos(2\pi \times 10^4 t)$, it is decreasing. Hence, the induced current has to be CW when looking down on the loop, as shown in the figure.