7.1 Write general expressions for the electric and magnetic fields of a 1 GHz sinusoidal plane wave traveling in the $+y$-direction in a lossless nonmagnetic medium with relative permittivity $\varepsilon_r = 9$. The electric field is polarized along the x-direction, its peak value is 6 V/m, and its intensity is 4 V/m at $t = 0$ and $y = 2$ cm.

Solution: For $f = 1$ GHz, $\mu_r = 1$, and $\varepsilon_r = 9$,

$$\omega = 2\pi f = 2\pi \times 10^9 \text{ rad/s},$$

$$k = \frac{2\pi}{\lambda} = \frac{2\pi}{\lambda_0}\sqrt{\varepsilon_r} = \frac{2\pi f}{c}\sqrt{\varepsilon_r} = \frac{2\pi \times 10^9}{3 \times 10^8}\sqrt{9} = 20\pi \text{ rad/m},$$

$$\mathbf{E}(y,t) = \hat{\mathbf{x}}6\cos(2\pi \times 10^9 t - 20\pi y + \phi_0) \qquad \text{(V/m)}.$$

At $t = 0$ and $y = 2$ cm, $E = 4$ V/m:

$$4 = 6\cos(-20\pi \times 2 \times 10^{-2} + \phi_0) = 6\cos(-0.4\pi + \phi_0).$$

Hence,

$$\phi_0 - 0.4\pi = \cos^{-1}\left(\frac{4}{6}\right) = 0.84 \text{ rad},$$

which gives

$$\phi_0 = 2.1 \text{ rad} = 120.19^\circ$$

and

$$\mathbf{E}(y,t) = \hat{\mathbf{x}}\,6\cos(2\pi \times 10^9 t - 20\pi y + 120.19^\circ) \qquad \text{(V/m)}.$$