7.12 The electric field of a uniform plane wave propagating in free space is given by

$$\widetilde{\mathbf{E}} = (\hat{\mathbf{x}} + j\hat{\mathbf{y}})20e^{-j\pi z/6} \qquad \text{(V/m)}$$

Specify the modulus and direction of the electric field intensity at the $z = 0$ plane at $t = 0$, 5, and 10 ns.

Solution:

$$\begin{aligned}
\mathbf{E}(z,t) &= \mathfrak{Re}[\widetilde{\mathbf{E}}e^{j\omega t}] \\
&= \mathfrak{Re}[(\hat{\mathbf{x}} + j\hat{\mathbf{y}})30e^{-j\pi z/6}e^{j\omega t}] \\
&= \mathfrak{Re}[(\hat{\mathbf{x}} + \hat{\mathbf{y}}e^{j\pi/2})30e^{-j\pi z/6}e^{j\omega t}] \\
&= \hat{\mathbf{x}}30\cos(\omega t - \pi z/6) + \hat{\mathbf{y}}30\cos(\omega t - \pi z/6 + \pi/2) \\
&= \hat{\mathbf{x}}30\cos(\omega t - \pi z/6) - \hat{\mathbf{y}}30\sin(\omega t - \pi z/6) \qquad \text{(V/m)},
\end{aligned}$$

$$|\mathbf{E}| = \left[E_x^2 + E_y^2\right]^{1/2} = 30 \qquad \text{(V/m)},$$

$$\psi = \tan^{-1}\left(\frac{E_y}{E_x}\right) = -(\omega t - \pi z/6).$$

From

$$f = \frac{c}{\lambda} = \frac{kc}{2\pi} = \frac{\pi/6 \times 3 \times 10^8}{2\pi} = 2.5 \times 10^7 \text{ Hz},$$

$$\omega = 2\pi f = 5\pi \times 10^7 \text{ rad/s}.$$

At $z = 0$,

$$\psi = -\omega t = -5\pi \times 10^7 t = \begin{cases} 0 & \text{at } t = 0, \\ -0.25\pi = -45^\circ & \text{at } t = 5 \text{ ns}, \\ -0.5\pi = -90^\circ & \text{at } t = 10 \text{ ns}. \end{cases}$$

Therefore, the wave is LHC polarized.