7.13 Compare the polarization states of each of the following pairs of plane waves:

(a) Wave 1: $\mathbf{E}_1 = \hat{\mathbf{x}}2\cos(\omega t - kz) + \hat{\mathbf{y}}2\sin(\omega t - kz)$.
Wave 2: $\mathbf{E}_2 = \hat{\mathbf{x}}2\cos(\omega t + kz) + \hat{\mathbf{y}}2\sin(\omega t + kz)$.

(b) Wave 1: $\mathbf{E}_1 = \hat{\mathbf{x}}2\cos(\omega t - kz) - \hat{\mathbf{y}}2\sin(\omega t - kz)$.
Wave 2: $\mathbf{E}_2 = \hat{\mathbf{x}}2\cos(\omega t + kz) - \hat{\mathbf{y}}2\sin(\omega t + kz)$.

Solution:

(a)

$$\begin{aligned}
\mathbf{E}_1 &= \hat{\mathbf{x}}2\cos(\omega t - kz) + \hat{\mathbf{y}}2\sin(\omega t - kz) \\
&= \hat{\mathbf{x}}2\cos(\omega t - kz) + \hat{\mathbf{y}}2\cos(\omega t - kz - \pi/2), \\
\widetilde{\mathbf{E}}_1 &= \hat{\mathbf{x}}2e^{-jkz} + \hat{\mathbf{y}}2e^{-jkz}e^{-j\pi/2}, \\
\psi_0 &= \tan^{-1}\left(\frac{ay}{ax}\right) = \tan^{-1} 1 = 45^\circ, \\
\delta &= -\pi/2.
\end{aligned}$$

Hence, wave 1 is RHC.

Similarly,

$$\widetilde{\mathbf{E}}_2 = \hat{\mathbf{x}}2e^{jkz} + \hat{\mathbf{y}}2e^{jkz}e^{-j\pi/2}.$$

Wave 2 has the same magnitude and phases as wave 1 except that its direction is along $-\hat{\mathbf{z}}$ instead of $+\hat{\mathbf{z}}$. Hence, the locus of rotation of $\mathbf{E}$ will match the left hand instead of the right hand. Thus, wave 2 is LHC.

(b)

$$\begin{aligned}
\mathbf{E}_1 &= \hat{\mathbf{x}}2\cos(\omega t - kz) - \hat{\mathbf{y}}2\sin(\omega t - kz), \\
\widetilde{\mathbf{E}}_1 &= \hat{\mathbf{x}}2e^{-jkz} + \hat{\mathbf{y}}2e^{-jkz}e^{j\pi/2}.
\end{aligned}$$

Wave 1 is LHC.

$$\widetilde{\mathbf{E}}_2 = \hat{\mathbf{x}}2e^{jkz} + \hat{\mathbf{y}}2e^{jkz}e^{j\pi/2}.$$

Reversal of direction of propagation (relative to wave 1) makes wave 2 RHC.