7.15 A linearly polarized plane wave of the form $\widetilde{\mathbf{E}} = \hat{\mathbf{x}} a_x e^{-jkz}$ can be expressed as the sum of an RHC polarized wave with magnitude a_R, and an LHC polarized wave with magnitude a_L. Prove this statement by finding expressions for a_R and a_L in terms of a_x.

Solution:

$$\widetilde{\mathbf{E}} = \hat{\mathbf{x}} a_x e^{-jkz},$$

$$\text{RHC wave:} \quad \widetilde{\mathbf{E}}_\mathrm{R} = a_\mathrm{R}(\hat{\mathbf{x}} + \hat{\mathbf{y}} e^{-j\pi/2}) e^{-jkz} = a_\mathrm{R}(\hat{\mathbf{x}} - j\hat{\mathbf{y}}) e^{-jkz},$$

$$\text{LHC wave:} \quad \widetilde{\mathbf{E}}_\mathrm{L} = a_\mathrm{L}(\hat{\mathbf{x}} + \hat{\mathbf{y}} e^{j\pi/2}) e^{-jkz} = a_\mathrm{L}(\hat{\mathbf{x}} + j\hat{\mathbf{y}}) e^{-jkz},$$

$$\widetilde{\mathbf{E}} = \widetilde{\mathbf{E}}_\mathrm{R} + \widetilde{\mathbf{E}}_\mathrm{L},$$

$$\hat{\mathbf{x}} a_x = a_\mathrm{R}(\hat{\mathbf{x}} - j\hat{\mathbf{y}}) + a_\mathrm{L}(\hat{\mathbf{x}} + j\hat{\mathbf{y}}).$$

By equating real and imaginary parts, $a_x = a_\mathrm{R} + a_\mathrm{L}$, $0 = -a_\mathrm{R} + a_\mathrm{L}$, or $a_\mathrm{L} = a_x/2$, $a_\mathrm{R} = a_x/2$.