**7.20** Ignoring reflection at the air-water boundary, if the amplitude of a 1 GHz incident wave in air is 20 V/m at the water surface, at what depth will it be down to 1 $\mu$V/m? Water has $\mu_r = 1$ and at 1 GHz, $\varepsilon_r = 80$ and $\sigma = 1$ S/m.

**Solution:** For water at 1 GHz,

$$\frac{\varepsilon''}{\varepsilon'} = \frac{\sigma}{\omega\varepsilon} = \frac{\sigma}{2\pi f \varepsilon_r \varepsilon_0} = \frac{1}{2\pi \times 10^9 \times 80 \times 8.85 \times 10^{-12}}$$
$$= 0.225.$$

Hence, it is neither a low-loss nor a conducting medium. Using Eq. (7.66a):

$$\alpha = \omega \left\{ \frac{\mu\varepsilon'}{2} \left[ \sqrt{1 + \left(\frac{\varepsilon''}{\varepsilon'}\right)^2} - 1 \right] \right\}^{1/2}$$
$$= \omega \left\{ \frac{\mu_r \varepsilon_r'}{2c^2} \left[ \sqrt{1 + \left(\frac{\varepsilon''}{\varepsilon'}\right)^2} - 1 \right] \right\}^{1/2}$$
$$= 2\pi \times 10^9 \left\{ \frac{80}{2(3 \times 10^8)} \left[ \sqrt{1 + (0.225)^2} - 1 \right] \right\}^{1/2}$$
$$= 20.94 \text{ Np/m.}$$

The depth $d$ at which $E$ is down to 1 $\mu$V/m is obtained from

$$10^{-6} = 20e^{-\alpha d} = 20e^{-20.94d},$$

which leads to

$$\ln\left(\frac{10^{-6}}{20}\right) = -20.94d,$$

or

$$d = \frac{6.39}{20.94} = 0.305 \text{ m} = 30.5 \text{ cm.}$$