7.25 The electric field of a plane wave propagating in a nonmagnetic medium is given by

$$\mathbf{E} = \hat{\mathbf{z}}\, 25 e^{-30x} \cos(2\pi \times 10^9 t - 40x) \qquad \text{(V/m)}$$

Obtain the corresponding expression for **H**.

Solution: From the given expression for **E**,

$$\omega = 2\pi \times 10^9 \qquad \text{(rad/s)},$$
$$\alpha = 30 \qquad \text{(Np/m)},$$
$$\beta = 40 \qquad \text{(rad/m)}.$$

From (7.65a) and (7.65b),

$$\alpha^2 - \beta^2 = -\omega^2 \mu \varepsilon' = -\omega^2 \mu_0 \varepsilon_0 \varepsilon_r' = -\frac{\omega^2}{c^2} \varepsilon_r',$$
$$2\alpha\beta = \omega^2 \mu \varepsilon'' = \frac{\omega^2}{c^2} \varepsilon_r''.$$

Using the above values for ω, α, and β, we obtain the following:

$$\varepsilon_r' = 1.6,$$
$$\varepsilon_r'' = 5.47.$$
$$\eta_c = \sqrt{\frac{\mu}{\varepsilon'}} \left(1 - j\frac{\varepsilon''}{\varepsilon'}\right)^{-1/2}$$
$$= \frac{\eta_0}{\sqrt{\varepsilon_r'}} \left(1 - j\frac{\varepsilon_r''}{\varepsilon_r'}\right)^{-1/2} = \frac{377}{\sqrt{1.6}} \left(1 - j\frac{5.47}{1.6}\right)^{-1/2} = 157.9 e^{j36.85^\circ} \qquad (\Omega).$$
$$\widetilde{\mathbf{E}} = \hat{\mathbf{z}}\, 25 e^{-30x} e^{-j40x},$$
$$\widetilde{\mathbf{H}} = \frac{1}{\eta_c} \hat{\mathbf{k}} \times \widetilde{\mathbf{E}} = \frac{1}{157.9 e^{j36.85^\circ}} \hat{\mathbf{x}} \times \hat{\mathbf{z}}\, 25 e^{-30x} e^{-j40x} = -\hat{\mathbf{y}}\, 0.16 e^{-30x} e^{-40x} e^{-j36.85^\circ},$$
$$\mathbf{H} = \Re\mathfrak{e}\{\widetilde{\mathbf{H}} e^{j\omega t}\} = -\hat{\mathbf{y}}\, 0.16 e^{-30x} \cos(2\pi \times 10^9 t - 40x - 36.85^\circ) \qquad \text{(A/m)}.$$