7.27 The magnetic field of a plane wave propagating in a nonmagnetic medium is given by

$$\mathbf{H} = \hat{\mathbf{y}}\, 60e^{-10z}\cos(2\pi \times 10^8 t - 12z) \qquad \text{(mA/m)}.$$

Obtain the corresponding expression for **E**.

Solution: From the expression for **H**,

$$f = 10^8 \text{ Hz}, \qquad \beta = 12 \text{ rad/m}, \qquad \alpha = 10 \text{ Np/m}.$$

From Eq. (7.65),

$$\varepsilon' = \frac{-(\alpha^2 - \beta^2)}{\omega^2 \mu} = \frac{-(100 - 144)}{(2\pi \times 10^8)^2 \times 4\pi \times 10^{-7}} = 8.87 \times 10^{-11} \text{ F/m},$$

$$\varepsilon'' = \frac{2\alpha\beta}{\omega^2 \mu} = \frac{2 \times 10 \times 12}{(2\pi \times 10^8)^2 \times 4\pi \times 10^{-7}} = 48.38 \times 10^{-11} \text{ F/m}.$$

Hence,

$$\begin{aligned}
\eta_c &= \sqrt{\frac{\mu}{\varepsilon'}}\left(1 - j\,\frac{\varepsilon''}{\varepsilon'}\right)^{-1/2} \\
&= \sqrt{\frac{4\pi \times 10^{-7}}{8.87 \times 10^{-11}}}\left(1 - j\,\frac{48.38 \times 10^{-11}}{8.87 \times 10^{-11}}\right)^{-1/2} \\
&= 119.03(1 - j5.45)^{-1/2} \\
&= 119.03(5.54e^{-j79.6^\circ})^{-1/2} = 119.03(0.42e^{j39.8^\circ}) = 50e^{j39.8^\circ}\ \Omega.
\end{aligned}$$