7.3 The electric field phasor of a uniform plane wave is given by $\widetilde{\mathbf{E}} = \hat{\mathbf{y}}\,10e^{j0.2z}$ (V/m). If the phase velocity of the wave is 1.5×10^8 m/s and the relative permeability of the medium is $\mu_r = 2.4$, find the following:

(a) The wavelength.

(b) The frequency f of the wave.

(c) The relative permittivity of the medium.

(d) The magnetic field $\mathbf{H}(z,t)$.

Solution:

(a) From $\widetilde{\mathbf{E}} = \hat{\mathbf{y}}10e^{j0.2z}$ (V/m), we deduce that $k = 0.2$ rad/m. Hence,

$$\lambda = \frac{2\pi}{k} = \frac{2\pi}{0.2} = 10\pi = 31.42 \text{ m}.$$

(b)

$$f = \frac{u_p}{\lambda} = \frac{1.5 \times 10^8}{31.42} = 4.77 \times 10^6 \text{ Hz} = 4.77 \text{ MHz}.$$

(c) From

$$u_p = \frac{c}{\sqrt{\mu_r \varepsilon_r}}, \qquad \varepsilon_r = \frac{1}{\mu_r}\left(\frac{c}{u_p}\right)^2 = \frac{1}{2.4}\left(\frac{3}{1.5}\right)^2 = 1.67.$$

(d)

$$\eta = \sqrt{\frac{\mu}{\varepsilon}} \simeq 120\pi\sqrt{\frac{\mu_r}{\varepsilon_r}} = 120\pi\sqrt{\frac{2.4}{1.67}} = 451.94 \qquad (\Omega),$$

$$\widetilde{\mathbf{H}} = \frac{1}{\eta}(-\hat{\mathbf{z}}) \times \widetilde{\mathbf{E}} = \frac{1}{\eta}(-\hat{\mathbf{z}}) \times \hat{\mathbf{y}}10e^{j0.2z} = \hat{\mathbf{x}}\,22.13e^{j0.2z} \qquad (\text{mA/m}),$$

$$\mathbf{H}(z,t) = \hat{\mathbf{x}}\,22.13\cos(\omega t + 0.2z) \qquad (\text{mA/m}),$$

with $\omega = 2\pi f = 9.54\pi \times 10^6$ rad/s.