7.39 Consider the imaginary rectangular box shown in **Fig.** P7.39.

(a) Determine the net power flux $P(t)$ entering the box due to a plane wave in air given by

$$\mathbf{E} = \hat{\mathbf{x}} E_0 \cos(\omega t - ky) \qquad \text{(V/m)}$$

(b) Determine the net time-average power entering the box.

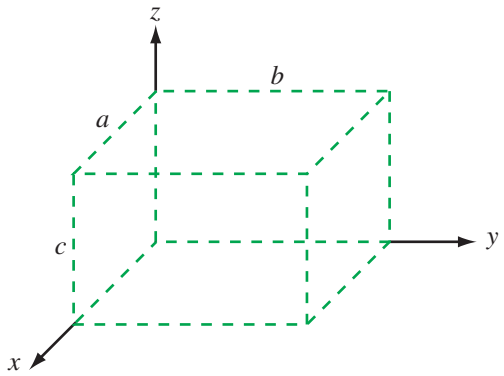

Figure P7.39 Imaginary rectangular box of Problems 7.39 and 7.40.

Solution:

(a)

$$\mathbf{E} = \hat{\mathbf{x}} E_0 \cos(\omega t - ky),$$

$$\mathbf{H} = -\hat{\mathbf{z}} \frac{E_0}{\eta_0} \cos(\omega t - ky).$$

$$\mathbf{S}(t) = \mathbf{E} \times \mathbf{H} = \hat{\mathbf{y}} \frac{E_0^2}{\eta_0} \cos^2(\omega t - ky),$$

$$P(t) = S(t)\, A|_{y=0} - S(t)\, A|_{y=b} = \frac{E_0^2}{\eta_0} ac[\cos^2 \omega t - \cos^2(\omega t - kb)].$$

(b)

$$P_{\text{av}} = \frac{1}{T} \int_0^T P(t)\, dt.$$

where $T = 2\pi/\omega$.

$$P_{\text{av}} = \frac{E_0^2 ac}{\eta_0} \left\{ \frac{\omega}{2\pi} \int_0^{2\pi/\omega} [\cos^2 \omega t - \cos^2(\omega t - kb)]\, dt \right\} = 0.$$

Net average energy entering the box is zero, which is as expected since the box is in a lossless medium (air).