

7.42 Given a wave with

$$\mathbf{E} = \hat{\mathbf{x}} E_0 \cos(\omega t - kz)$$

calculate:

(a) The time-average electric energy density

$$(w_e)_{\text{av}} = \frac{1}{T} \int_0^T w_e dt = \frac{1}{2T} \int_0^T \epsilon E^2 dt$$

(b) The time-average magnetic energy density

$$(w_m)_{\text{av}} = \frac{1}{T} \int_0^T w_m dt = \frac{1}{2T} \int_0^T \mu H^2 dt$$

(c) Show that $(w_e)_{\text{av}} = (w_m)_{\text{av}}$.

Solution:

(a)

$$(w_e)_{\text{av}} = \frac{1}{2T} \int_0^T \epsilon E_0^2 \cos^2(\omega t - kz) dt.$$

With $T = \frac{2\pi}{\omega}$,

$$\begin{aligned} (w_e)_{\text{av}} &= \frac{\omega \epsilon E_0^2}{4\pi} \int_0^{2\pi/\omega} \cos^2(\omega t - kz) dt \\ &= \frac{\epsilon E_0^2}{4\pi} \int_0^{2\pi} \cos^2(\omega t - kz) d(\omega t) \\ &= \frac{\epsilon E_0^2}{4}. \end{aligned}$$

(b)

$$\mathbf{H} = \hat{\mathbf{y}} \frac{E_0}{\eta} \cos(\omega t - kz).$$

$$\begin{aligned} (w_m)_{\text{av}} &= \frac{1}{2T} \int_0^T \mu H^2 dt \\ &= \frac{1}{2T} \int_0^T \mu \frac{E_0^2}{\eta^2} \cos^2(\omega t - kz) dt \\ &= \frac{\mu E_0^2}{4\eta^2}. \end{aligned}$$

(c)

$$(w_m)_{\text{av}} = \frac{\mu E_0^2}{4\eta^2} = \frac{\mu E_0^2}{4\left(\frac{\mu}{\epsilon}\right)} = \frac{\epsilon E_0^2}{4} = (w_e)_{\text{av}}.$$
