

7.6 The magnetic field of a plane wave propagating in a nonmagnetic material is given by

$$\mathbf{H} = \hat{\mathbf{x}}60\cos(2\pi \times 10^7 t + 0.1\pi y) \\ \hat{\mathbf{z}}30\cos(2\pi \times 10^7 t + 0.1\pi y) \quad (\text{mA/m}).$$

Determine

- (a) The wavelength.
- (b) ϵ_r .
- (c) \mathbf{E} .

Solution:

- (a) From the expression for \mathbf{H} , we deduce that

$$\omega = 2\pi \times 10^7 \text{ rad/s} \rightarrow f = 10^7 \text{ Hz}, \\ k = 0.1\pi \text{ rad/m} \rightarrow \lambda = \frac{2\pi}{k} = 20 \text{ m}.$$

- (b)

$$u_p = f\lambda = 10^7 \times 20 = 2 \times 10^8 \text{ m/s}, \\ \epsilon_r = \left(\frac{c}{u_p}\right)^2 = \left(\frac{3 \times 10^8}{2 \times 10^8}\right)^2 = (1.5)^2 = 2.25.$$

- (c) From Eq. (7.39b),

$$\mathbf{E} = -\eta \hat{\mathbf{k}} \times \mathbf{H}.$$

Here,

$$\eta = \frac{\eta_0}{\sqrt{\epsilon_r}} = \frac{377}{\sqrt{2.25}} = 251.3 \, \Omega, \\ \hat{\mathbf{k}} = -\hat{\mathbf{y}}.$$

Hence,

$$\mathbf{E} = -251.3\hat{\mathbf{y}} \times [\hat{\mathbf{x}}60\cos(2\pi \times 10^7 t + 0.1\pi y) + \hat{\mathbf{z}}30\cos(2\pi \times 10^7 t + 0.1\pi y)] \times 10^{-3} \\ = \hat{\mathbf{z}}15.08\cos(2\pi \times 10^7 t + 0.1\pi y) - \hat{\mathbf{x}}7.54\cos(2\pi \times 10^7 t + 0.1\pi y) \quad (\text{V/m}).$$
