7.7 The electric field of a plane wave propagating in a lossless, nonmagnetic, dielectric material with $\varepsilon_r = 2.56$ is given by

$$\mathbf{E} = \hat{\mathbf{y}}\, 5\cos(6\pi \times 10^9 t - kz) \qquad \text{(V/m)}$$

Determine:

(a) f, u_p, λ, k, and η.

(b) The magnetic field $\mathbf{H}$.

Solution:

(a)

$$\omega = 2\pi f = 6\pi \times 10^9 \text{ rad/s},$$

$$f = 3 \times 10^9 \text{ Hz} = 3 \text{ GHz},$$

$$u_p = \frac{c}{\sqrt{\varepsilon_r}} = \frac{3 \times 10^8}{\sqrt{2.56}} = 1.875 \times 10^8 \text{ m/s},$$

$$\lambda = \frac{u_p}{f} = \frac{1.875 \times 10^8}{3 \times 10^9} = 6.24 \text{ cm},$$

$$k = \frac{2\pi}{\lambda} = \frac{2\pi}{6.24 \times 10^{-2}} = 100.8 \text{ rad/m},$$

$$\eta = \frac{\eta_0}{\sqrt{\varepsilon_r}} = \frac{377}{\sqrt{2.56}} = \frac{377}{1.6} = 235.62\ \Omega.$$

(b)

$$\mathbf{H} = -\hat{\mathbf{x}}\frac{5}{\eta}\cos(6\pi \times 10^9 t - kz)$$

$$= -\hat{\mathbf{x}}\frac{5}{235.62}\cos(6\pi \times 10^9 t - 100.8z)$$

$$= -\hat{\mathbf{x}}\, 2.12 \times 10^{-2}\cos(6\pi \times 10^9 t - 100.8z) \qquad \text{(A/m)}.$$