8.10 For the configuration shown in Fig. P8.9, use transmission-line equations (or the Smith chart) to calculate the input impedance at $z = -d$ for $\varepsilon_{r_1} = 1$, $\varepsilon_{r_2} = 9$, $\varepsilon_{r_3} = 4$, $d = 1.2$ m, and $f = 50$ MHz. Also determine the fraction of the incident average power density reflected by the structure. Assume all media are lossless and nonmagnetic.

Solution: In medium 2,

$$\lambda = \frac{\lambda_0}{\sqrt{\varepsilon_{r_2}}} = \frac{c}{f\sqrt{\varepsilon_{r_2}}} = \frac{3\times 10^8}{5\times 10^7 \times 3} = 2 \text{ m}.$$

Hence,

$$\beta_2 = \frac{2\pi}{\lambda_2} = \pi \text{ rad/m}, \qquad \beta_2 d = 1.2\pi \text{ rad}.$$

At $z = -d$, the input impedance of a transmission line with load impedance Z_L is given by Eq. (2.63) as

$$Z_{in}(-d) = Z_0\left(\frac{Z_L + jZ_0 \tan\beta_2 d}{Z_0 + jZ_L \tan\beta_2 d}\right).$$

In the present case, $Z_0 = \eta_2 = \eta_0/\sqrt{\varepsilon_{r_2}} = \eta_0/3$ and $Z_L = \eta_3 = \eta_0/\sqrt{\varepsilon_{r_3}} = \eta_0/2$, where $\eta_0 = 120\pi$ (Ω). Hence,

$$Z_{in}(-d) = \eta_2\left(\frac{\eta_3 + j\eta_2 \tan\beta_2 d}{\eta_2 + j\eta_3 \tan\beta_2 d}\right) = \frac{\eta_0}{3}\left(\frac{\frac{1}{2} + j\left(\frac{1}{3}\right)\tan 1.2\pi}{\frac{1}{3} + j\left(\frac{1}{2}\right)\tan 1.2\pi}\right) = \eta_0(0.35 - j0.14).$$

At $z = -d$,

$$\Gamma = \frac{Z_{in} - Z_1}{Z_{in} + Z_1} = \frac{\eta_0(0.35 - j0.14) - \eta_0}{\eta_0(0.35 - j0.14) + \eta_0} = 0.49e^{-j162.14^\circ}.$$

Fraction of incident power reflected by the structure is $|\Gamma|^2 = |0.49|^2 = 0.24$.