8.14 Consider a thin film of soap in air under illumination by yellow light with $\lambda = 0.6\ \mu$m in vacuum. If the film is treated as a planar dielectric slab with $\varepsilon_r = 1.72$, surrounded on both sides by air, what film thickness would produce strong reflection of the yellow light at normal incidence?

Solution: The transmission line analogue of the soap-bubble wave problem is shown in **Fig.** P8.14(b) where the load Z_L is equal to η_0, the impedance of the air medium on the other side of the bubble. That is,

$$\eta_0 = 377\ \Omega, \qquad \eta_1 = \frac{377}{\sqrt{1.72}} = 287.5\ \Omega.$$

The normalized load impedance is

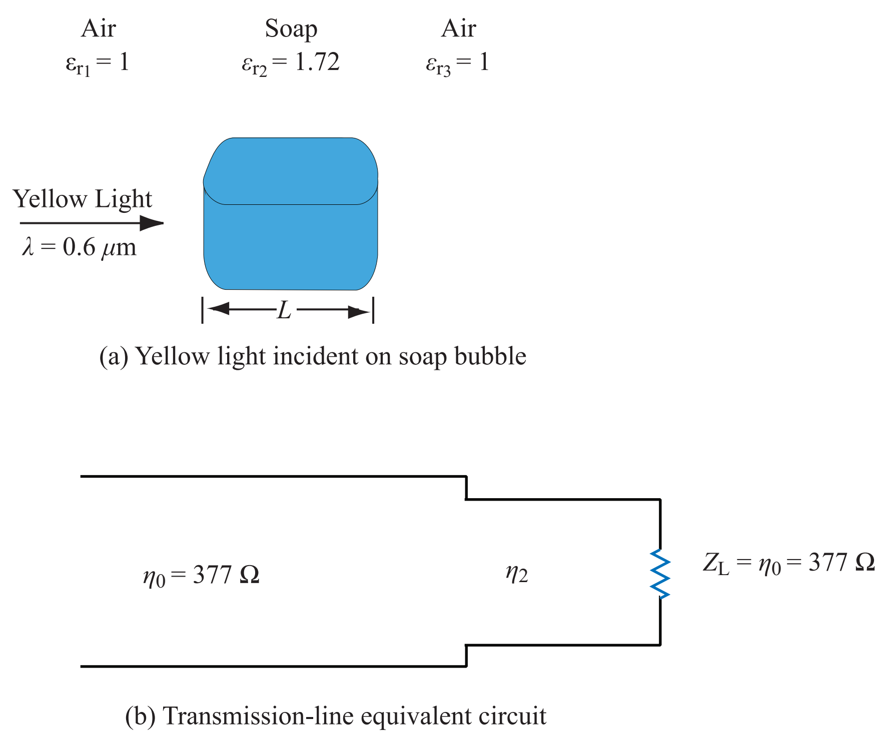

Figure P8.14 Diagrams for Problem 8.14.

$$z_L = \frac{\eta_0}{\eta_1} = 1.31.$$

For the reflection by the soap bubble to be the largest, Z_{in} needs to be the most different from η_0. This happens when z_{L} is transformed through a length $\lambda/4$. Hence,

$$L = \frac{\lambda}{4} = \frac{\lambda_0}{4\sqrt{\varepsilon_r}} = \frac{0.6\ \mu\text{m}}{4\sqrt{1.72}} = 0.115\ \mu\text{m},$$

where λ is the wavelength of the soap bubble material. Strong reflections will also occur if the thickness is greater than L by integer multiples of $n\lambda/2 = (0.23n)\ \mu$m.

Hence, in general

$$L = (0.115 + 0.23n)\ \mu\text{m}, \qquad n = 0, 1, 2, \ldots\ .$$

According to Section 2-7.5, transforming a load $Z_{\text{L}} = 377\ \Omega$ through a $\lambda/4$ section of $Z_0 = 287.5\ \Omega$ ends up presenting an input impedance of

$$Z_{\text{in}} = \frac{Z_0^2}{Z_{\text{L}}} = \frac{(287.5)^2}{377} = 219.25\ \Omega.$$

This Z_{in} is at the input side of the soap bubble. The reflection coefficient at that interface is

$$\Gamma = \frac{Z_{\text{in}} - \eta_0}{Z_{\text{in}} + \eta_0} = \frac{219.25 - 377}{219.25 + 377} = -0.27.$$

Any other thickness would produce a reflection coeffficient with a smaller magnitude.