8.28 Repeat Problem 8.27 for a wave in air with

$$\widetilde{\mathbf{H}}^{\mathrm{i}} = \hat{\mathbf{y}}\, 2 \times 10^{-2} e^{-j(8x+6z)} \qquad \text{(A/m)}$$

incident upon the planar boundary of a dielectric medium ($z \geq 0$) with $\varepsilon_{\mathrm{r}} = 9$.

Solution:

(a) $\widetilde{\mathbf{H}}^{\mathrm{i}} = \hat{\mathbf{y}}\, 2 \times 10^{-2} e^{-j(8x+6z)}$.

Since $\mathbf{H}^{\mathrm{i}}$ is along $\hat{\mathbf{y}}$, which is perpendicular to the plane of incidence, the wave is TM polarized, or equivalently, its electric field vector is parallel polarized (parallel to the plane of incidence).

(b) From Eq. (8.65b), the argument of the exponential is

$$-jk_1(x \sin\theta_{\mathrm{i}} + z \cos\theta_{\mathrm{i}}) = -j(8x + 6z).$$

Hence,

$$k_1 \sin\theta_{\mathrm{i}} = 8, \qquad k_1 \cos\theta_{\mathrm{i}} = 6,$$

from which we determine

$$\theta_{\mathrm{i}} = \tan^{-1}\left(\frac{8}{6}\right) = 53.13^\circ,$$

$$k_1 = \sqrt{6^2 + 8^2} = 10 \qquad \text{(rad/m)}.$$

Also,

$$\omega = u_{\mathrm{p}} k = ck = 3 \times 10^8 \times 10 = 3 \times 10^9 \qquad \text{(rad/s)}.$$

(c)

$$\eta_1 = \eta_0 = 377\ \Omega,$$

$$\eta_2 = \frac{\eta_0}{\sqrt{\varepsilon_{\mathrm{r}_2}}} = \frac{\eta_0}{3} = 125.67\ \Omega,$$

$$\theta_{\mathrm{t}} = \sin^{-1}\left[\frac{\sin\theta_{\mathrm{i}}}{\sqrt{\varepsilon_{\mathrm{r}_2}}}\right] = \sin^{-1}\left[\frac{\sin 53.13^\circ}{\sqrt{9}}\right] = 15.47^\circ,$$

$$\Gamma_{\parallel} = \frac{\eta_2 \cos\theta_{\mathrm{t}} - \eta_1 \cos\theta_{\mathrm{i}}}{\eta_2 \cos\theta_{\mathrm{t}} + \eta_1 \cos\theta_{\mathrm{i}}} = -0.30,$$

$$\tau_{\parallel} = (1 + \Gamma_{\parallel}) \frac{\cos\theta_{\mathrm{i}}}{\cos\theta_{\mathrm{t}}} = 0.44.$$

In accordance with Eqs. (8.65a) to (8.65d), $E_0^{\mathrm{i}} = 2 \times 10^{-2} \eta_1$ and

$$\widetilde{\mathbf{E}}^{\mathrm{i}} = (\hat{\mathbf{x}} \cos\theta_{\mathrm{i}} - \hat{\mathbf{z}} \sin\theta_{\mathrm{i}})\, 2 \times 10^{-2} \eta_1\, e^{-j(8x+6z)} = (\hat{\mathbf{x}} 4.52 - \hat{\mathbf{z}} 6.03)\, e^{-j(8x+6z)}.$$

$\widetilde{\mathbf{E}}^{\mathrm{r}}$ is similar to $\widetilde{\mathbf{E}}^{\mathrm{i}}$ except for reversal of z-components and multiplication of amplitude by $\Gamma_{\parallel}$. Hence, with $\Gamma_{\parallel} = -0.30$,

$$\begin{aligned}
\mathbf{E}^{\mathrm{r}} &= \mathfrak{Re}[\widetilde{\mathbf{E}}^{\mathrm{r}} e^{j\omega t}] = -(\hat{\mathbf{x}}\, 1.36 + \hat{\mathbf{z}}\, 1.81) \cos(3 \times 10^9 t - 8x + 6z) \text{ V/m}, \\
\mathbf{H}^{\mathrm{r}} &= \hat{\mathbf{y}}\, 2 \times 10^{-2} \Gamma_{\parallel} \cos(3 \times 10^9 t - 8x + 6z) \\
&= -\hat{\mathbf{y}}\, 0.6 \times 10^{-2} \cos(3 \times 10^9 t - 8x + 6z) \text{ A/m}.
\end{aligned}$$

(d) In medium 2,

$$\begin{aligned}
k_2 &= k_1 \sqrt{\frac{\varepsilon_2}{\varepsilon_1}} = 10\sqrt{9} = 30 \text{ rad/m}, \\
\theta_{\mathrm{t}} &= \sin^{-1} \left[\sqrt{\frac{\varepsilon_2}{\varepsilon_1}} \sin \theta_{\mathrm{i}} \right] = \sin^{-1} \left[\frac{1}{3} \sin 53.13^\circ \right] = 15.47^\circ,
\end{aligned}$$

and the exponent of $\mathbf{E}^{\mathrm{t}}$ and $\mathbf{H}^{\mathrm{t}}$ is

$$-jk_2(x \sin \theta_{\mathrm{t}} + z \cos \theta_{\mathrm{t}}) = -j30(x \sin 15.47^\circ + z \cos 15.47^\circ) = -j(8x + 28.91\, z).$$

Hence,

$$\begin{aligned}
\widetilde{\mathbf{E}}^{\mathrm{t}} &= (\hat{\mathbf{x}} \cos \theta_{\mathrm{t}} - \hat{\mathbf{z}} \sin \theta_{\mathrm{t}}) E_0^{\mathrm{i}} \tau_{\parallel} e^{-j(8x+28.91\, z)} \\
&= (\hat{\mathbf{x}}\, 0.96 - \hat{\mathbf{z}}\, 0.27)\, 2 \times 10^{-2} \times 377 \times 0.44\, e^{-j(8x+28.91\, z)} \\
&= (\hat{\mathbf{x}}\, 3.18 - \hat{\mathbf{z}}\, 0.90)\, e^{-j(8x+28.91\, z)}, \\
\widetilde{\mathbf{H}}^{\mathrm{t}} &= \hat{\mathbf{y}} \frac{E_0^{\mathrm{i}} \tau_{\parallel}}{\eta_2}\, e^{-j(8x+28.91\, z)} \\
&= \hat{\mathbf{y}}\, 2.64 \times 10^{-2}\, e^{-j(8x+28.91\, z)}, \\
\mathbf{E}^{\mathrm{t}} &= \mathfrak{Re}\{\widetilde{\mathbf{E}}^{\mathrm{t}} e^{j\omega t}\} \\
&= (\hat{\mathbf{x}}\, 3.18 - \hat{\mathbf{z}}\, 0.90) \cos(3 \times 10^9 t - 8x - 28.91\, z) \text{ V/m}, \\
\mathbf{H}^{\mathrm{t}} &= \hat{\mathbf{y}}\, 2.64 \times 10^{-2} \cos(3 \times 10^9 t - 8x - 28.91\, z) \text{ A/m}.
\end{aligned}$$

(e)

$$S_{\mathrm{av}}^{\mathrm{t}} = \frac{|E_0^{\mathrm{t}}|^2}{2\eta_2} = \frac{|H_0^{\mathrm{t}}|^2}{2} \eta_2 = \frac{(2.64 \times 10^{-2})^2}{2} \times 125.67 = 44 \text{ mW/m}^2.$$