

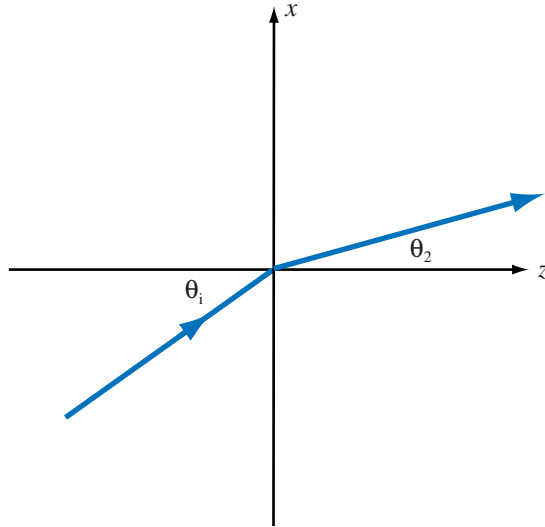
**8.29** A plane wave in air with

$$\tilde{\mathbf{E}}^i = (\hat{\mathbf{x}}9 - \hat{\mathbf{y}}4 - \hat{\mathbf{z}}6)e^{-j(2x+3z)} \quad (\text{V/m})$$

is incident upon the planar surface of a dielectric material, with  $\epsilon_r = 2.25$ , occupying the half-space  $z \geq 0$ . Determine

- (a) The incidence angle  $\theta_i$ .
- (b) The frequency of the wave.
- (c) The field  $\tilde{\mathbf{E}}^r$  of the reflected wave.
- (d) The field  $\tilde{\mathbf{E}}^t$  of the wave transmitted into the dielectric medium.
- (e) The average power density carried by the wave into the dielectric medium.

**Solution:**



(a) From the exponential of the given expression, it is clear that the wave direction of travel is in the  $x$ - $z$  plane. By comparison with the expressions in (8.48a) for perpendicular polarization or (8.65a) for parallel polarization, both of which have the same phase factor, we conclude that:

$$k_1 \sin \theta_i = 2,$$

$$k_1 \cos \theta_i = 3.$$

Hence,

$$k_1 = \sqrt{2^2 + 3^2} = 3.6 \quad (\text{rad/m})$$

$$\theta_1 = \tan^{-1}(2/3) = 33.7^\circ.$$

Also,

$$k_2 = k_1 \sqrt{\epsilon_{r2}} = 3.6 \sqrt{2.25} = 5.4 \quad (\text{rad/m})$$

$$\theta_2 = \sin^{-1} \left[ \sin \theta_1 \sqrt{\frac{1}{2.25}} \right] = 21.7^\circ.$$

(b)

$$k_1 = \frac{2\pi f}{c}$$

$$f = \frac{k_1 c}{2\pi} = \frac{3.6 \times 3 \times 10^8}{2\pi} = 172 \text{ MHz}.$$

(c) In order to determine the electric field of the reflected wave, we first have to determine the polarization of the wave. The vector argument in the given expression for  $\tilde{\mathbf{E}}^i$  indicates that the incident wave is a mixture of parallel and perpendicular polarization components. Perpendicular polarization has a  $\hat{\mathbf{y}}$ -component only (see 8.46a), whereas parallel polarization has only  $\hat{\mathbf{x}}$  and  $\hat{\mathbf{z}}$  components (see 8.65a). Hence, we shall decompose the incident wave accordingly:

$$\tilde{\mathbf{E}}^i = \tilde{\mathbf{E}}_{\perp}^i + \tilde{\mathbf{E}}_{\parallel}^i$$

with

$$\tilde{\mathbf{E}}_{\perp}^i = -\hat{\mathbf{y}} 4 e^{-j(2x+3z)} \quad (\text{V/m})$$

$$\tilde{\mathbf{E}}_{\parallel}^i = (\hat{\mathbf{x}} 9 - \hat{\mathbf{z}} 6) e^{-j(2x+3z)} \quad (\text{V/m})$$

From the above expressions, we deduce:

$$E_{\perp 0}^i = -4 \text{ V/m}$$

$$E_{\parallel 0}^i = \sqrt{9^2 + 6^2} = 10.82 \text{ V/m}.$$

Next, we calculate  $\Gamma$  and  $\tau$  for each of the two polarizations:

$$\Gamma_{\perp} = \frac{\cos \theta_1 - \sqrt{(\epsilon_2/\epsilon_1) - \sin^2 \theta_1}}{\cos \theta_1 + \sqrt{(\epsilon_2/\epsilon_1) - \sin^2 \theta_1}}$$

Using  $\theta_1 = 33.7^\circ$  and  $\epsilon_2/\epsilon_1 = 2.25/1 = 2.25$  leads to:

$$\Gamma_{\perp} = -0.25$$

$$\tau_{\perp} = 1 + \Gamma_{\perp} = 0.75.$$

Similarly,

$$\Gamma_{\perp} = \frac{-(\epsilon_2/\epsilon_1) \cos \theta_i + \sqrt{(\epsilon_2/\epsilon_1) - \sin^2 \theta_i}}{(\epsilon_2/\epsilon_1) \cos \theta_i + \sqrt{(\epsilon_2/\epsilon_1) - \sin^2 \theta_i}} = -0.15,$$

$$\tau_{\parallel} = (1 + \Gamma_{\parallel}) \frac{\cos \theta_i}{\cos \theta_t} = (1 - 0.15) \frac{\cos 33.7^\circ}{\cos 21.7^\circ} = 0.76.$$

The electric fields of the reflected and transmitted waves for the two polarizations are given by (8.49a), (8.49c), (8.65c), and (8.65e):

$$\begin{aligned}\tilde{\mathbf{E}}_{\perp}^r &= \hat{\mathbf{y}} E_{\perp 0}^r e^{-jk_1(x \sin \theta_r - z \cos \theta_r)} \\ \tilde{\mathbf{E}}_{\perp}^t &= \hat{\mathbf{y}} E_{\perp 0}^t e^{-jk_2(x \sin \theta_t + z \cos \theta_t)} \\ \tilde{\mathbf{E}}_{\parallel}^r &= (\hat{\mathbf{x}} \cos \theta_r + \hat{\mathbf{z}} \sin \theta_r) E_{\parallel 0}^r e^{-jk_1(x \sin \theta_r - z \cos \theta_r)} \\ \tilde{\mathbf{E}}_{\parallel}^t &= (\hat{\mathbf{x}} \cos \theta_t - \hat{\mathbf{z}} \sin \theta_t) E_{\parallel 0}^t e^{-jk_2(x \sin \theta_t + z \cos \theta_t)}\end{aligned}$$

Based on our earlier calculations:

$$\begin{aligned}\theta_r &= \theta_i = 33.7^\circ \\ \theta_t &= 21.7^\circ \\ k_1 &= 3.6 \text{ rad/m}, \quad k_2 = 5.4 \text{ rad/m}, \\ E_{\perp 0}^r &= \Gamma_{\perp} E_{\perp 0}^i = (-0.25) \times (-4) = 1 \text{ V/m.} \\ E_{\perp 0}^t &= \tau_{\perp} E_{\perp 0}^i = 0.75 \times (-4) = -3 \text{ V/m.} \\ E_{\parallel 0}^r &= \Gamma_{\parallel} E_{\parallel 0}^i = (-0.15) \times 10.82 = -1.62 \text{ V/m.} \\ E_{\parallel 0}^t &= \tau_{\parallel} E_{\parallel 0}^i = 0.76 \times 10.82 = 8.22 \text{ V/m.}\end{aligned}$$

Using the above values, we have:

$$\begin{aligned}\tilde{\mathbf{E}}^r &= \tilde{\mathbf{E}}_{\perp}^r + \tilde{\mathbf{E}}_{\parallel}^r \\ &= (\hat{\mathbf{x}} E_{\parallel 0}^r \cos \theta_r + \hat{\mathbf{y}} E_{\perp 0}^r + \hat{\mathbf{z}} E_{\parallel 0}^r \sin \theta_r) e^{-j(2x-3z)} \\ &= (-\hat{\mathbf{x}} 1.35 + \hat{\mathbf{y}} - \hat{\mathbf{z}} 0.90) e^{-j(2x-3z)} \quad (\text{V/m}).\end{aligned}$$

(d)

$$\tilde{\mathbf{E}}^t = \tilde{\mathbf{E}}_{\perp}^t + \tilde{\mathbf{E}}_{\parallel}^t$$

$$= (\hat{\mathbf{x}}7.65 - \hat{\mathbf{y}}3 - \hat{\mathbf{z}}3.05)e^{-j(2x+5z)} \quad (\text{V/m}).$$

(e)

$$S^t = \frac{|E_0^t|^2}{2\eta_2}$$

$$|E_0^t|^2 = (7.65)^2 + 3^2 + (3.05)^2 = 76.83$$

$$\eta_2 = \frac{\eta_0}{\sqrt{\epsilon_{r_2}}} = \frac{377}{1.5} = 251.3 \, \Omega$$

$$S^t = \frac{76.83}{2 \times 251.3} = 152.86 \quad (\text{mW/m}^2).$$


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