

8.36 A 50-MHz right-hand circularly polarized plane wave with an electric field modulus of 30 V/m is normally incident in air upon a dielectric medium with $\epsilon_r = 9$ and occupying the region defined by $z \geq 0$.

- (a) Write an expression for the electric field phasor of the incident wave, given that the field is a positive maximum at $z = 0$ and $t = 0$.
- (b) Calculate the reflection and transmission coefficients.
- (c) Write expressions for the electric field phasors of the reflected wave, the transmitted wave, and the total field in the region $z \leq 0$.
- (d) Determine the percentages of the incident average power reflected by the boundary and transmitted into the second medium.

Solution:

(a)

$$k_1 = \frac{\omega}{c} = \frac{2\pi \times 50 \times 10^6}{3 \times 10^8} = \frac{\pi}{3} \text{ rad/m},$$

$$k_2 = \frac{\omega}{c} \sqrt{\epsilon_{r2}} = \frac{\pi}{3} \sqrt{9} = \pi \text{ rad/m}.$$

From (7.57), RHC wave traveling in $+z$ direction:

$$\begin{aligned} \tilde{\mathbf{E}}^i &= a_0(\hat{\mathbf{x}} + \hat{\mathbf{y}} e^{-j\pi/2}) e^{-jk_1 z} = a_0(\hat{\mathbf{x}} - j\hat{\mathbf{y}}) e^{-jk_1 z} \\ \mathbf{E}^i(z, t) &= \Re \left[\tilde{\mathbf{E}}^i e^{j\omega t} \right] \\ &= \Re \left[a_0(\hat{\mathbf{x}} e^{j(\omega t - k_1 z)} + \hat{\mathbf{y}} e^{j(\omega t - k_1 z - \pi/2)}) \right] \\ &= \hat{\mathbf{x}} a_0 \cos(\omega t - k_1 z) + \hat{\mathbf{y}} a_0 \cos(\omega t - k_1 z - \pi/2) \\ &= \hat{\mathbf{x}} a_0 \cos(\omega t - k_1 z) + \hat{\mathbf{y}} a_0 \sin(\omega t - k_1 z) \\ |\mathbf{E}^i| &= [a_0^2 \cos^2(\omega t - k_1 z) + a_0^2 \sin^2(\omega t - k_1 z)]^{1/2} = a_0 = 30 \text{ V/m}. \end{aligned}$$

Hence,

$$\tilde{\mathbf{E}}^i = 30(x_0 - jy_0) e^{-j\pi z/3} \quad (\text{V/m}).$$

(b)

$$\begin{aligned} \eta_1 &= \eta_0 = 120\pi \quad (\Omega), \quad \eta_2 = \frac{\eta_0}{\sqrt{\epsilon_{r2}}} = \frac{120\pi}{\sqrt{9}} = 40\pi \quad (\Omega). \\ \Gamma &= \frac{\eta_2 - \eta_1}{\eta_2 + \eta_1} = \frac{40\pi - 120\pi}{40\pi + 120\pi} = -0.5 \\ \tau &= 1 + \Gamma = 1 - 0.5 = 0.5. \end{aligned}$$

(c)

$$\begin{aligned}\tilde{\mathbf{E}}^r &= \Gamma a_0 (\hat{\mathbf{x}} - j\hat{\mathbf{y}}) e^{jk_1 z} \\ &= -0.5 \times 30 (\hat{\mathbf{x}} - j\hat{\mathbf{y}}) e^{jk_1 z} \\ &= -15 (\hat{\mathbf{x}} - j\hat{\mathbf{y}}) e^{j\pi z/3} \quad (\text{V/m}). \\ \tilde{\mathbf{E}}^t &= \tau a_0 (\hat{\mathbf{x}} - j\hat{\mathbf{y}}) e^{-jk_2 z} \\ &= 15 (\hat{\mathbf{x}} - j\hat{\mathbf{y}}) e^{-j\pi z} \quad (\text{V/m}). \\ \tilde{\mathbf{E}}_1 &= \tilde{\mathbf{E}}^i + \tilde{\mathbf{E}}^r \\ &= 30 (\hat{\mathbf{x}} - j\hat{\mathbf{y}}) e^{-j\pi z/3} - 15 (\hat{\mathbf{x}} - j\hat{\mathbf{y}}) e^{j\pi z/3} \\ &= 15 (\hat{\mathbf{x}} - j\hat{\mathbf{y}}) [2e^{-j\pi z/3} - e^{j\pi z/3}] \quad (\text{V/m}).\end{aligned}$$

(d)

$$\% \text{ of reflected power} = 100 \times |\Gamma|^2 = 100 \times (0.5)^2 = 25\%$$

$$\% \text{ of transmitted power} = 100 |\tau|^2 \frac{\eta_1}{\eta_2} = 100 \times (0.5)^2 \times \frac{120\pi}{40\pi} = 75\%.$$
