8.38 A TE wave propagating in a dielectric-filled waveguide of unknown permittivity has dimensions $a = 5$ cm and $b = 3$ cm. If the x-component of its electric field is given by

$$\begin{aligned} E_x = -36\cos(40\pi x)\sin(100\pi y) \\ \cdot \sin(2.4\pi \times 10^{10} t - 52.9\pi z), \qquad \text{(V/m)} \end{aligned}$$

determine:

(a) the mode number,

(b) ε_r of the material in the guide,

(c) the cutoff frequency, and

(d) the expression for H_y.

Solution:

(a) Comparison of the given expression with Eq. (8.110a) reveals that

$$\frac{m\pi}{a} = 40\pi, \qquad \text{hence } m = 2$$

$$\frac{n\pi}{b} = 100\pi, \qquad \text{hence } n = 3.$$

Mode is TE_{23}.

(b) From $\sin(\omega t - \beta z)$, we deduce that

$$\omega = 2.4\pi \times 10^{10} \text{ rad/s}, \qquad \beta = 52.9\pi \text{ rad/m}.$$

Using Eq. (8.105) to solve for ε_r, we have

$$\begin{aligned} \varepsilon_r &= \frac{c^2}{\omega^2}\left[\beta^2 + \left(\frac{m\pi}{a}\right)^2 + \left(\frac{n\pi}{b}\right)^2\right] \\ &= 2.25. \end{aligned}$$

(c)

$$u_{p_0} = \frac{c}{\sqrt{\varepsilon_r}} = \frac{3 \times 10^8}{\sqrt{2.25}} = 2 \times 10^8 \text{ m/s}.$$

$$\begin{aligned} f_{23} &= \frac{u_{p_0}}{2}\sqrt{\left(\frac{2}{a}\right)^2 + \left(\frac{3}{b}\right)^2} \\ &= 10.77 \text{ GHz}. \end{aligned}$$

8.38 A TE wave propagating in a dielectric-filled waveguide of unknown permittivity has dimensions $a = 5$ cm and $b = 3$ cm. If the x-component of its electric field is given by

$$E_x = -36\cos(40\pi x)\sin(100\pi y) \cdot \sin(2.4\pi \times 10^{10} t - 52.9\pi z), \qquad \text{(V/m)}$$

determine:

(a) the mode number,

(b) ε_r of the material in the guide,

(c) the cutoff frequency, and

(d) the expression for H_y.

Solution:

(a) Comparison of the given expression with Eq. (8.110a) reveals that

$$\frac{m\pi}{a} = 40\pi, \qquad \text{hence } m = 2$$
$$\frac{n\pi}{b} = 100\pi, \qquad \text{hence } n = 3.$$

Mode is TE_{23}.

(b) From $\sin(\omega t - \beta z)$, we deduce that

$$\omega = 2.4\pi \times 10^{10} \text{ rad/s}, \qquad \beta = 52.9\pi \text{ rad/m}.$$

Using Eq. (8.105) to solve for ε_r, we have

$$\varepsilon_r = \frac{c^2}{\omega^2}\left[\beta^2 + \left(\frac{m\pi}{a}\right)^2 + \left(\frac{n\pi}{b}\right)^2\right] = 2.25.$$

(c)

$$u_{p_0} = \frac{c}{\sqrt{\varepsilon_r}} = \frac{3 \times 10^8}{\sqrt{2.25}} = 2 \times 10^8 \text{ m/s}.$$

$$f_{23} = \frac{u_{p_0}}{2}\sqrt{\left(\frac{2}{a}\right)^2 + \left(\frac{3}{b}\right)^2} = 10.77 \text{ GHz}.$$