8.4 A 200-MHz, left-hand circularly polarized plane wave with an electric field modulus of 5 V/m is normally incident in air upon a dielectric medium with $\varepsilon_r = 4$, and occupies the region defined by $z \geq 0$.

(a) Write an expression for the electric field phasor of the incident wave, given that the field is a positive maximum at $z = 0$ and $t = 0$.

(b) Calculate the reflection and transmission coefficients.

(c) Write expressions for the electric field phasors of the reflected wave, the transmitted wave, and the total field in the region $z \leq 0$.

(d) Determine the percentages of the incident average power reflected by the boundary and transmitted into the second medium.

Solution:

(a)

$$k_1 = \frac{\omega}{c} = \frac{2\pi \times 2 \times 10^8}{3 \times 10^8} = \frac{4\pi}{3} \text{ rad/m},$$

$$k_2 = \frac{\omega}{u_{p_2}} = \frac{\omega}{c}\sqrt{\varepsilon_{r_2}} = \frac{4\pi}{3}\sqrt{4} = \frac{8\pi}{3} \text{ rad/m}.$$

LHC wave:

$$\widetilde{\mathbf{E}}^i = a_0(\hat{\mathbf{x}} + \hat{\mathbf{y}}e^{j\pi/2})e^{-jkz} = a_0(\hat{\mathbf{x}} + j\hat{\mathbf{y}})e^{-jkz},$$

$$\mathbf{E}^i(z,t) = \hat{\mathbf{x}}a_0\cos(\omega t - kz) - \hat{\mathbf{y}}a_0\sin(\omega t - kz),$$

$$|\mathbf{E}^i| = [a_0^2\cos^2(\omega t - kz) + a_0^2\sin^2(\omega t - kz)]^{1/2} = a_0 = 5 \qquad \text{(V/m)}.$$

Hence,

$$\widetilde{\mathbf{E}}^i = 5(\hat{\mathbf{x}} + j\hat{\mathbf{y}})e^{-j4\pi z/3} \qquad \text{(V/m)}.$$

(b)

$$\eta_1 = \eta_0 = 120\pi \quad (\Omega), \qquad \eta_2 = \frac{\eta_0}{\sqrt{\varepsilon_r}} = \frac{\eta_0}{2} = 60\pi \quad (\Omega).$$

Equations (8.8a) and (8.9) give

$$\Gamma = \frac{\eta_2 - \eta_1}{\eta_2 + \eta_1} = \frac{60\pi - 120\pi}{60\pi + 120\pi} = \frac{-60}{180} = -\frac{1}{3}, \qquad \tau = 1 + \Gamma = \frac{2}{3}.$$

(c)

$$\widetilde{\mathbf{E}}^r = 5\Gamma(\hat{\mathbf{x}} + j\hat{\mathbf{y}})e^{jk_1 z} = -\frac{5}{3}(\hat{\mathbf{x}} + j\hat{\mathbf{y}})e^{j4\pi z/3} \qquad \text{(V/m)},$$

$$\widetilde{\mathbf{E}}^t = 5\tau(\hat{\mathbf{x}} + j\hat{\mathbf{y}})e^{-jk_2 z} = \frac{10}{3}(\hat{\mathbf{x}} + j\hat{\mathbf{y}})e^{-j8\pi z/3} \qquad \text{(V/m)},$$

$$\widetilde{\mathbf{E}}_1 = \widetilde{\mathbf{E}}^{\mathrm{i}} + \widetilde{\mathbf{E}}^{\mathrm{r}} = 5(\hat{\mathbf{x}} + j\hat{\mathbf{y}}) \left[e^{-j4\pi z/3} - \frac{1}{3} e^{j4\pi z/3} \right] \qquad \text{(V/m)}.$$

(d)

$$\%\text{ of reflected power} = 100 \times |\Gamma|^2 = \frac{100}{9} = 11.11\%,$$

$$\%\text{ of transmitted power} = 100 \times |\tau|^2 \frac{\eta_1}{\eta_2} = 100 \times \left(\frac{2}{3}\right)^2 \times \frac{120\pi}{60\pi} = 88.89\%.$$