8.40 Derive Eq. (8.89b).

Solution:

We start with Eqs. (8.88a and e),

$$\frac{\partial \widetilde{e}_z}{\partial y} + j\beta \widetilde{e}_y = -j\omega\mu \widetilde{h}_x,$$

$$-j\beta \widetilde{h}_x - \frac{\partial \widetilde{h}_z}{\partial x} = j\omega\varepsilon \widetilde{e}_y.$$

To eliminate $\widetilde{h}_x$, we multiply the top equation by β and the bottom equation by $\omega\mu$, and then we add them together. The result is:

$$\beta \frac{\partial \widetilde{e}_z}{\partial y} + j\beta^2 \widetilde{e}_y - \omega\mu \frac{\partial \widetilde{h}_z}{\partial x} = j\omega^2\mu\varepsilon \widetilde{e}_y.$$

Multiplying all terms by $e^{-j\beta z}$ to convert $\widetilde{e}_y$ to $\widetilde{E}_y$ (and similarly for the other field components), and then solving for $\widetilde{E}_y$ leads to

$$\begin{aligned}
\widetilde{E}_y &= \frac{1}{j(\beta^2 - \omega^2\mu\varepsilon)} \left(-\beta \frac{\partial \widetilde{E}_z}{\partial y} + \omega\mu \frac{\partial \widetilde{H}_z}{\partial x} \right) \\
&= \frac{j}{k_c^2} \left(-\beta \frac{\partial \widetilde{E}_z}{\partial y} + \omega\mu \frac{\partial \widetilde{H}_z}{\partial x} \right),
\end{aligned}$$

where we used the relation

$$k_c^2 = \omega^2\mu\varepsilon - \beta^2.$$