8.44 For a rectangular waveguide operating in the TE_{10} mode, obtain expressions for the surface charge density $\widetilde{\rho}_s$ and surface current density $\widetilde{\mathbf{J}}_s$ on each of the four walls of the guide.

Solution:

For TE_{10}, the expressions for $\widetilde{\mathbf{E}}$ and $\widetilde{\mathbf{H}}$ are given by Eq. (8.110) with $m = 1$ and $n = 0$,

$$\widetilde{E}_x = 0,$$

$$\widetilde{E}_y = -j\frac{\omega\mu\pi H_0}{k_c^2 a}\sin\left(\frac{\pi x}{a}\right)e^{-j\beta z},$$

$$\widetilde{E}_z = 0,$$

$$\widetilde{H}_x = j\frac{\beta\pi H_0}{k_c^2 a}\sin\left(\frac{\pi x}{a}\right)e^{-j\beta z},$$

$$\widetilde{H}_y = 0,$$

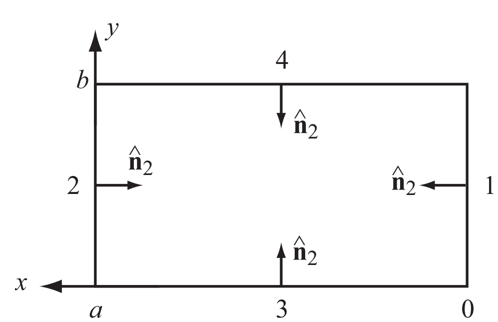
$$\widetilde{H}_z = H_0\cos\left(\frac{\pi x}{a}\right)e^{-j\beta z}.$$

The applicable boundary conditions are given in Table 6-2. At the boundary between a dielectric (medium 1) and a conductor (medium 2),

$$\widetilde{\rho}_s = \hat{\mathbf{n}}_2\cdot\widetilde{\mathbf{D}}_1 = \varepsilon_1\,\hat{\mathbf{n}}_2\cdot\widetilde{\mathbf{E}}_1,$$

$$\widetilde{\mathbf{J}}_s = \hat{\mathbf{n}}_2\times\widetilde{\mathbf{H}}_1,$$

where $\widetilde{\mathbf{E}}_1$ and $\widetilde{\mathbf{H}}_1$ are the fields inside the guide, ε_1 is the permittivity of the material filling the guide, and $\hat{\mathbf{n}}_2$ is the normal to the guide wall, pointing away from the wall (inwardly). In view of the coordinate system defined for the guide, $\hat{\mathbf{n}}_2 = \hat{\mathbf{x}}$ for side wall at $x = 0$, $\hat{\mathbf{n}}_2 = -\hat{\mathbf{x}}$ for wall at $x = a$, etc.



(a) At side wall 1 at $x = 0$, $\hat{\mathbf{n}}_2 = \hat{\mathbf{x}}$. Hence,

$$\begin{aligned}
\rho_s &= \varepsilon_1 \hat{\mathbf{x}} \cdot \hat{\mathbf{y}} E_y|_{x=0} = 0 \\
\mathbf{J}_s &= \hat{\mathbf{x}} \times (\hat{\mathbf{x}} \widetilde{H}_x + \hat{\mathbf{z}} \widetilde{H}_z)|_{x=0} \\
&= -\hat{\mathbf{y}} \widetilde{H}_z|_{x=0} \\
&= -\hat{\mathbf{y}} H_0 e^{-j\beta z}.
\end{aligned}$$

(b) At side wall 2 at $x = a$, $\hat{\mathbf{n}}_2 = -\hat{\mathbf{x}}$. Hence,

$$\begin{aligned}
\rho_s &= 0 \\
\mathbf{J}_s &= \hat{\mathbf{y}} H_0 e^{-j\beta z}.
\end{aligned}$$

(c) At bottom surface at $y = 0$, $\hat{\mathbf{n}}_2 = \hat{\mathbf{y}}$. Hence,

$$\begin{aligned}
\rho_s &= \varepsilon_1 \hat{\mathbf{y}} \cdot \hat{\mathbf{y}} E_y|_{y=0} \\
&= -j \frac{\omega \varepsilon \mu \pi H_0}{k_c^2 a} \sin\left(\frac{\pi x}{a}\right) e^{-j\beta z} \\
\widetilde{\mathbf{J}}_s &= \hat{\mathbf{y}} \times (\hat{\mathbf{x}} \widetilde{H}_x + \hat{\mathbf{z}} \widetilde{H}_z) \\
&= H_0 \left[\hat{\mathbf{x}} \cos\left(\frac{\pi x}{a}\right) - \hat{\mathbf{z}}\, j \frac{\beta \pi}{k_c^2 a} \sin\left(\frac{\pi x}{a}\right)\right] e^{-j\beta z}.
\end{aligned}$$

(d) At top surface at $y - b$, $\hat{\mathbf{n}}_2 = -\hat{\mathbf{y}}$. Hence,

$$\begin{aligned}
\widetilde{\rho}_s &= j \frac{\omega \varepsilon \mu \pi H_0}{k_c^2 a} \sin\left(\frac{\pi x}{a}\right) e^{-j\beta z} \\
\widetilde{\mathbf{J}}_s &= H_0 \left[-\hat{\mathbf{x}} \cos\left(\frac{\pi x}{a}\right) + \hat{\mathbf{z}}\, j \frac{\beta \pi}{k_c^2 a} \sin\left(\frac{\pi x}{a}\right)\right] e^{-j\beta z}.
\end{aligned}$$