8.5 Repeat Problem 8.4, but replace the dielectric medium with a poor conductor characterized by $\varepsilon_r = 2.25$, $\mu_r = 1$, and $\sigma = 10^{-4}$ S/m.

Solution:

(a) Medium 1:

$$\eta_1 = \eta_0 = 120\pi \quad (\Omega), \qquad k_1 = \frac{\omega}{c} = \frac{2\pi \times 2 \times 10^8}{3 \times 10^8} = \frac{4\pi}{3} \qquad \text{(rad/m)}.$$

Medium 2:

$$\frac{\sigma_2}{\omega \varepsilon_2} = \frac{10^{-4} \times 36\pi}{2\pi \times 2 \times 10^8 \times 2.25 \times 10^{-9}} = 4 \times 10^{-3}.$$

Hence, medium 2 is a low-loss dielectric. From Table 7-1,

$$\begin{aligned}
\alpha_2 &= \frac{\sigma_2}{2}\sqrt{\frac{\mu_2}{\varepsilon_2}} \\
&= \frac{\sigma_2}{2}\frac{120\pi}{\sqrt{\varepsilon_{r_2}}} = \frac{\sigma_2}{2} \times \frac{120\pi}{\sqrt{2.25}} = \frac{10^{-4}}{2} \times \frac{120\pi}{1.5} = 1.26 \times 10^{-2} \qquad \text{(NP/m)}, \\
\beta_2 &= \omega\sqrt{\mu_2 \varepsilon_2} = \frac{\omega\sqrt{\varepsilon_{r_2}}}{c} = 2\pi \qquad \text{(rad/m)}, \\
\eta_2 &= \sqrt{\frac{\mu_2}{\varepsilon_2}}\left(1 + \frac{j\sigma_2}{2\omega\varepsilon_2}\right) = \frac{120\pi}{\sqrt{\varepsilon_{r_2}}}\left(1 + j2 \times 10^{-3}\right) \simeq \frac{120\pi}{1.5} = 80\pi \quad (\Omega).
\end{aligned}$$

LHC wave:

$$\begin{aligned}
\widetilde{\mathbf{E}}^{\mathrm{i}} &= a_0(\hat{\mathbf{x}} + j\hat{\mathbf{y}})e^{-jk_1 z}, \\
|\widetilde{\mathbf{E}}^{\mathrm{i}}| &= a_0 = 5 \qquad \text{(V/m)}, \\
\widetilde{\mathbf{E}}^{\mathrm{i}} &= 5(\hat{\mathbf{x}} + j\hat{\mathbf{y}})e^{-j4\pi z/3} \qquad \text{(V/m)}.
\end{aligned}$$

(b) According to Eqs. (8.8a) and (8.9),

$$\Gamma = \frac{\eta_2 - \eta_1}{\eta_2 + \eta_1} = \frac{80\pi - 120\pi}{80\pi + 120\pi} = -0.2, \qquad \tau = 1 + \Gamma = 1 - 0.2 = 0.8.$$

(c)

$$\begin{aligned}
\widetilde{\mathbf{E}}^{\mathrm{r}} &= 5\Gamma(\hat{\mathbf{x}} + j\hat{\mathbf{y}})e^{jk_1 z} = -(\hat{\mathbf{x}} + j\hat{\mathbf{y}})e^{j4\pi z/3} \qquad \text{(V/m)}, \\
\widetilde{\mathbf{E}}^{\mathrm{t}} &= 5\tau(\hat{\mathbf{x}} + j\hat{\mathbf{y}})e^{-\alpha_2 z}e^{-j\beta_z z} = 4(\hat{\mathbf{x}} + j\hat{\mathbf{y}})e^{-1.26\times 10^{-2} z}e^{-j2\pi z} \qquad \text{(V/m)}, \\
\widetilde{\mathbf{E}}_1 &= \widetilde{\mathbf{E}}^{\mathrm{i}} + \widetilde{\mathbf{E}}^{\mathrm{r}} = 5(\hat{\mathbf{x}} + j\hat{\mathbf{y}})[e^{-j4\pi z/3} - 0.2e^{j4\pi z/3}] \qquad \text{(V/m)}.
\end{aligned}$$

(d)

$$\% \text{ of reflected power} = 100|\Gamma|^2 = 100(0.2)^2 = 4\%,$$

$$\% \text{ of transmitted power} = 100|\tau|^2 \frac{\eta_1}{\eta_2} = 100(0.8)^2 \times \frac{120\pi}{80\pi} = 96\%.$$