8.6 A 50-MHz plane wave with electric field amplitude of 50 V/m is normally incident in air onto a semi-infinite, perfect dielectric medium with $\varepsilon_r = 36$. Determine the following:

(a) Γ

(b) The average power densities of the incident and reflected waves.

(c) The distance in the air medium from the boundary to the nearest minimum of the electric field intensity, $|\mathbf{E}|$.

Solution:

(a)

$$\eta_1 = \eta_0 = 120\pi \quad (\Omega), \qquad \eta_2 = \sqrt{\frac{\mu_2}{\varepsilon_2}} = \frac{120\pi}{\sqrt{\varepsilon_{r_2}}} = \frac{120\pi}{6} = 20\pi \quad (\Omega),$$

$$\Gamma = \frac{\eta_2 - \eta_1}{\eta_2 + \eta_1} = \frac{20\pi - 120\pi}{20\pi + 120\pi} = -0.71.$$

Hence, $|\Gamma| = 0.71$ and $\theta_\eta = 180°$.

(b)

$$S^{\rm i}_{\rm av} = \frac{|E^{\rm i}_0|^2}{2\eta_1} = \frac{(50)^2}{2 \times 120\pi} = 3.32 \qquad ({\rm W/m^2}),$$

$$S^{\rm r}_{\rm av} = |\Gamma|^2 S^{\rm i}_{\rm av} = (0.71)^2 \times 3.32 = 1.67 \qquad ({\rm W/m^2}).$$

(c) In medium 1 (air),

$$\lambda_1 = \frac{c}{f} = \frac{3 \times 10^8}{5 \times 10^7} = 6 \text{ m}.$$

From Eqs. (8.16) and (8.17),

$$l_{\max} = \frac{\theta_r \lambda_1}{4\pi} = \frac{\pi \times 6}{4\pi} = 1.5 \text{ m},$$

$$l_{\min} = l_{\max} - \frac{\lambda_1}{4} = 1.5 - 1.5 = 0 \text{ m (at the boundary)}.$$