8.7 Repeat Problem 8.6, but replace the dielectric medium with a conductor with $\varepsilon_r = 1$, $\mu_r = 1$, and $\sigma = 2.78 \times 10^{-3}$ S/m.

Solution:

(a) Medium 1:

$$\eta_1 = \eta_0 = 120\pi = 377 \quad (\Omega), \qquad \lambda_1 = \frac{c}{f} = \frac{3 \times 10^8}{5 \times 10^7} = 6 \text{ m},$$

Medium 2:

$$\frac{\sigma_2}{\omega\varepsilon_2} = \frac{2.78 \times 10^{-3} \times 36\pi}{2\pi \times 5 \times 10^7 \times 10^{-9}} = 1.$$

Hence, Medium 2 is a quasi-conductor. From Eq. (7.70),

$$\begin{aligned}\eta_2 = \sqrt{\frac{\mu_2}{\varepsilon_2}}\left(1 - j\frac{\varepsilon_2''}{\varepsilon_2'}\right)^{-1/2} &= 120\pi\left(1 - j\frac{\sigma_2}{\omega\varepsilon_2}\right)^{-1/2} \\ &= 120\pi(1 - j1)^{-1/2} \\ &= 120\pi(\sqrt{2})^{-1/2}e^{j22.5^\circ} = (292.88 + j121.31) \qquad (\Omega).\end{aligned}$$

$$\Gamma = \frac{\eta_2 - \eta_1}{\eta_2 + \eta_1} = \frac{(292.88 + j121.31) - 377}{(292.88 + j121.31) + 377} = -0.09 + j0.12 = 0.22\angle 114.5^\circ.$$

(b)

$$S^{\text{i}}_{\text{av}} = \frac{|E^{\text{i}}_0|^2}{2\eta_1} = \frac{50^2}{2 \times 120\pi} = 3.32 \qquad (\text{W/m}^2),$$

$$|S^{\text{r}}_{\text{av}}| = |\Gamma|^2 S^{\text{i}}_{\text{av}} = (0.22)^2(3.32) = 0.16 \qquad (\text{W/m}^2).$$

(c) In medium 1 (air),

$$\lambda_1 = \frac{c}{f} = \frac{3 \times 10^8}{5 \times 10^7} = 6 \text{ m}.$$

For $\theta_r = 114.5^\circ = 2$ rad, Eqs. (8.16) and (8.17) give

$$l_{\max} = \frac{\theta_r \lambda_1}{4\pi} + \frac{(0)\lambda_1}{2} = \frac{2(6)}{4} + 0 = 3 \text{ m},$$

$$l_{\min} = l_{\max} - \frac{\lambda_1}{4} = 3 - \frac{6}{4} = 3 - 1.5 = 1.5 \text{ m}.$$