9.21 A car antenna is a vertical monopole over a conducting surface. Repeat Problem 9.5 for a 1-m–long car antenna operating at 1 MHz. The antenna wire is made of aluminum with $\mu_c = \mu_0$ and $\sigma_c = 3.5 \times 10^7$ S/m, and its diameter is 1 cm.

Solution:

(a) Following Example 9-3, $\lambda = c/f = \left(3 \times 10^8 \text{ m/s}\right) / \left(10^6 \text{ Hz}\right) = 300$ m. As $l/\lambda = 2 \times (1 \text{ m}) / (300 \text{ m}) = 0.0067$, this antenna is a short (Hertzian) monopole. From Section 9-3.3, the radiation resistance of a monopole is half that for a corresponding dipole. Thus,

$$R_{\text{rad}} = \tfrac{1}{2} 80\pi^2 \left(\frac{l}{\lambda}\right)^2 = 40\pi^2 (0.0067)^2 = 17.7 \qquad (\text{m}\Omega),$$

$$R_{\text{loss}} = \frac{l}{2\pi a}\sqrt{\frac{\pi f \mu_c}{\sigma_c}} = \frac{1 \text{ m}}{\pi (10^{-2} \text{ m})}\sqrt{\frac{\pi (10^6 \text{ Hz})(4\pi \times 10^{-7} \text{ H/m})}{3.5 \times 10^7 \text{ S/m}}} = 10.7 \text{ m}\Omega,$$

$$\xi = \frac{R_{\text{rad}}}{R_{\text{rad}} + R_{\text{loss}}} = \frac{17.7 \text{ m}\Omega}{17.7 \text{ m}\Omega + 10.7 \text{ m}\Omega} = 62\%.$$

(b) From Example 9-2, a Hertzian dipole has a directivity of 1.5. The gain, from Eq. (9.29), is $G = \xi D = 0.62 \times 1.5 = 0.93 = -0.3$ dB.

(c) From Eq. (9.30a),

$$I_0 = \sqrt{\frac{2P_{\text{rad}}}{R_{\text{rad}}}} = \sqrt{\frac{2(80 \text{ W})}{17.7 \text{ m}\Omega}} = 95 \text{ A},$$

and from Eq. (9.31),

$$P_t = \frac{P_{\text{rad}}}{\xi} = \frac{80 \text{ W}}{0.62} = 129.2 \text{ W}.$$