9.27 The configuration shown in Fig. P9.27 depicts two vertically oriented half-wave dipole antennas pointed towards each other, with both positioned on 100 m tall towers separated by a distance of 5 km. If the transit antenna is driven by a 50 MHz current with amplitude $I_0 = 2$ A, determine:

(a) The power received by the receive antenna in the absence of the surface. (Assume both antennas to be lossless.)

(b) The power received by the receive antenna after incorporating reflection by the ground surface, assuming the surface to be flat and to have $\varepsilon_r = 9$ and conductivity $\sigma = 10^{-3}$ (S/m).

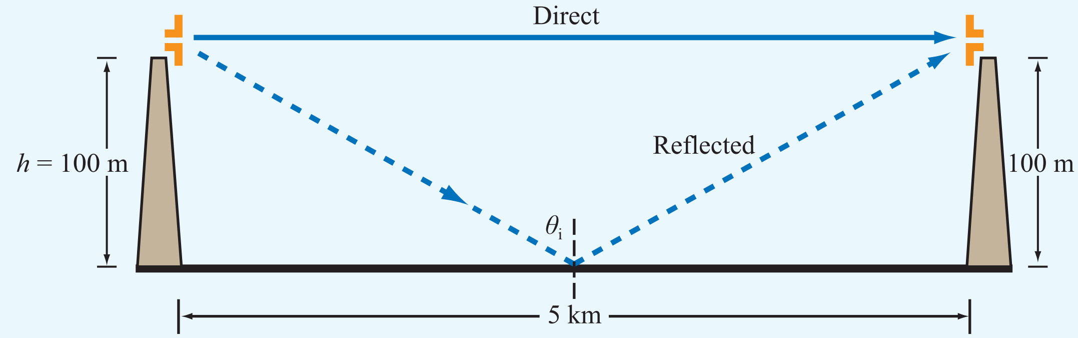

Figure P9.27 Problem 9.27.

Solution:

(a) Since both antennas are lossless,

$$P_{\text{rec}} = P_{\text{int}} = S_i A_{\text{er}}$$

where S_i is the incident power density and A_{er} is the effective area of the receive dipole. From Section 9-3,

$$S_i = S_0 = \frac{15I_0^2}{\pi R^2} ,$$

and from (9.64) and (9.47),

$$A_{\text{er}} = \frac{\lambda^2 D}{4\pi} = \frac{\lambda^2}{4\pi} \times 1.64 = \frac{1.64\lambda^2}{4\pi} .$$

Hence,

$$P_{\text{rec}} = \frac{15I_0^2}{\pi R^2} \times \frac{1.64\lambda^2}{4\pi} = 3.6 \times 10^{-6} \text{ W}.$$

(b) The electric field of the signal intercepted by the receive antenna now consists of a direct component, E_d, due to the directly transmitted signal, and a reflected component, E_r, due to the ground reflection. Since the power density S and the electric field E are related by

$$S = \frac{|E|^2}{2\eta_0},$$

it follows that

$$\begin{aligned} E_\text{d} &= \sqrt{2\eta_0 S_\text{i}}\; e^{-jkR} \\ &= \sqrt{2\eta_0 \times \frac{15 I_0^2}{\pi R^2}}\; e^{-jkR} \\ &= \sqrt{\frac{30\eta_0}{\pi}}\; \frac{I_0}{R}\; e^{-jkR} \end{aligned}$$

where the phase of the signal is measured with respect to the location of the transmit antenna, and $k = 2\pi/\lambda$. Hence,

$$E_\text{d} = 0.024 e^{-j120^\circ} \qquad \text{(V/m)}.$$

The electric field of the reflected signal is similar in form except for the fact that R should be replaced with R', where R' is the path length traveled by the reflected signal, and the electric field is modified by the reflection coefficient Γ. Thus,

$$E_\text{r} = \left(\sqrt{\frac{30\eta_0}{\pi}}\; \frac{I_0}{R'}\; e^{-jkR'} \right) \Gamma.$$

From the problem geometry

$$R' = 2\sqrt{(2.5 \times 10^3)^2 + (100)^2} = 5004.0 \text{ m}.$$

Since the dipole is vertically oriented, the electric field is parallel polarized. To calculate Γ, we first determine

$$\frac{\varepsilon''}{\varepsilon'} = \frac{\sigma}{\omega \varepsilon_0 \varepsilon_\text{r}} = \frac{10^{-3}}{2\pi \times 50 \times 10^6 \times 8.85 \times 10^{-12} \times 9} = 0.04.$$

From Table 7-1,

$$\eta_\text{c} \approx \eta = \sqrt{\frac{\mu}{\varepsilon}} = \frac{\eta_0}{\sqrt{\varepsilon_\text{r}}} = \frac{\eta_0}{\sqrt{9}} = \frac{\eta_0}{3}.$$

From (8.66a),

$$\Gamma_\parallel = \frac{\eta_2 \cos\theta_\text{t} - \eta_1 \cos\theta_\text{i}}{\eta_2 \cos\theta_\text{t} + \eta_1 \cos\theta_\text{i}}$$

From the geometry,

$$\cos\theta_\mathrm{i} = \frac{h}{(R'/2)} = \frac{100}{2502} = 0.04$$

$$\theta_\mathrm{i} = 87.71^\circ$$

$$\theta_\mathrm{t} = \sin^{-1}\left(\frac{\sin\theta_\mathrm{i}}{\sqrt{\varepsilon_\mathrm{r}}}\right) = 19.46^\circ$$

$$\eta_1 = \eta_0 \text{ (air)}$$

$$\eta_2 = \eta = \frac{\eta_0}{3} \, .$$

Hence,

$$\Gamma_\| = \frac{(\eta_0/3)\times 0.94 - \eta_0 \times 0.04}{(\eta_0/3)\times 0.94 + \eta_0 \times 0.04} = 0.77.$$

The reflected electric field is

$$E_\mathrm{r} = \left(\sqrt{\frac{30\eta_0}{\pi}}\,\frac{I_0}{R'}\,e^{-jkR'}\right)\Gamma$$

$$= 0.018e^{j0.6^\circ} \qquad \text{(V/m)}.$$

The total electric field is

$$E = E_\mathrm{d} + E_\mathrm{r}$$

$$= 0.024e^{-j120^\circ} + 0.018e^{j0.6^\circ}$$

$$= 0.02e^{-j73.3^\circ} \qquad \text{(V/m)}.$$

The received power is

$$P_\mathrm{rec} = S_\mathrm{i} A_\mathrm{er}$$

$$= \frac{|E|^2}{2\eta_0} \times \frac{1.64\lambda^2}{4\pi}$$

$$= 2.5 \times 10^{-6} \text{ W}.$$