9.30 The 10 dB beamwidth is the beam size between the angles at which $F(\theta)$ is 10 dB below its peak value. Determine the 10 dB beamwidth in the x–z plane for a uniformly illuminated aperture with length $l_x = 10\lambda$.

Solution: For a uniformly illuminated antenna of length $l_x = 10\lambda$ Eq. (9.90) gives

$$F(\theta) = \text{sinc}^2(\pi l_x \sin\theta/\lambda) = \text{sinc}^2(10\pi \sin\theta).$$

The peak value of $F(\theta)$ is 1, and the 10 dB level below the peak corresponds to when $F(\theta) = 0.1$ (because $10\log 0.1 = -10$ dB). Hence, we set $F(\theta) = 0.1$ and solve for θ:

$$0.1 = \text{sinc}^2(10\pi \sin\theta).$$

From tabulated values of the sinc function, it follows that the solution of this equation is

$$10\pi \sin\theta = 2.319$$

or

$$\theta \approx 4.23^\circ.$$

Hence, the 10-dB beamwidth is

$$\beta \approx 2\theta = 8.46^\circ.$$