9.5 Repeat Problem 9.4 for an antenna with

$$F(\theta,\phi) = \begin{cases} \sin^2\theta\cos^2\phi & \text{for } 0 \le \theta \le \pi \\ & \text{and } -\pi/2 \le \phi \le \pi/2 \\ 0 & \text{elsewhere} \end{cases}$$

Solution: The direction of maximum radiation is the $+\hat{\mathbf{x}}$ axis (where $\theta = \pi/2$ and $\phi = 0$). From Eq. (9.23),

$$\begin{aligned} D &= \frac{4\pi}{\iint_{4\pi} F \, d\Omega} \\ &= \frac{4\pi}{\int_{-\pi/2}^{\pi/2} \int_0^{\pi} \sin^2\theta \cos^2\phi \sin\theta \, d\theta \, d\phi} \\ &= \frac{4\pi}{\int_{-\pi/2}^{\pi/2} \cos^2\phi d\phi \int_0^{\pi} \sin^3\theta \, d\theta} \\ &= \frac{4\pi}{\int_{-\pi/2}^{\pi/2} \frac{1}{2}(1+\cos 2\phi) \, d\phi \int_{-1}^{1} \left(1-x^2\right) \, dx} \\ &= \frac{4\pi}{\frac{1}{2}\left(\phi + \frac{1}{2}\sin 2\phi\right)\Big|_{-\pi/2}^{\pi/2} \left(x - x^3/3\right)\Big|_{-1}^{1}} = \frac{4\pi}{\frac{1}{2}\pi\,(4/3)} = 6 = 7.8 \text{ dB}, \\ \Omega_{\text{p}} &= \frac{4\pi \text{ sr}}{D} = \frac{4\pi \text{ sr}}{6} = \frac{2}{3}\pi \qquad \text{(sr)}. \end{aligned}$$

In the x-z plane, $\phi = 0$ and the half power beamwidth is 90°, since

$$\sin^2(45^\circ) = \sin^2(135^\circ) = \tfrac{1}{2} \,.$$